

Quadratic Theory

Completing the Square

2001 P1	4. Given $f(x) = x^2 + 2x - 8$, express $f(x)$ in the form $(x + a)^2 - b$.	2
2003 P1	2. (a) Write $f(x) = x^2 + 6x + 11$ in the form $(x + a)^2 + b$. (b) Hence or otherwise sketch the graph of $y = f(x)$.	2 2
2002 P1	7. (a) Express $f(x) = x^2 - 4x + 5$ in the form $f(x) = (x - a)^2 + b$. (b) On the same diagram sketch: (i) the graph of $y = f(x)$; (ii) the graph of $y = 10 - f(x)$. (c) Find the range of values of x for which $10 - f(x)$ is positive.	2 4 1
2006 P1	8. (a) Express $2x^2 + 4x - 3$ in the form $a(x + b)^2 + c$. (b) Write down the coordinates of the turning point on the parabola with equation $y = 2x^2 + 4x - 3$.	3 1
2013 P1	21. Express $2x^2 + 12x + 1$ in the form $a(x + b)^2 + c$.	3
2015 OLD P2	2. Functions f and g are defined on suitable domains by $f(x) = 10 + x \text{ and } g(x) = (1 + x)(3 - x) + 2.$ (a) Find an expression for $f(g(x))$. (b) Express $f(g(x))$ in the form $p(x + q)^2 + r$. (c) Another function h is given by $h(x) = \frac{1}{f(g(x))}$. What values of x cannot be in the domain of h ?	2 3 2
2015 SP P1	8. $f(x)$ and $g(x)$ are functions, defined on the set of real numbers, such that $f(x) = 1 - \frac{1}{2}x \text{ and } g(x) = 8x^2 - 3.$ (a) Given that $h(x) = g(f(x))$, show that $h(x) = 2x^2 - 8x + 5$. (b) Express $h(x)$ in the form $a(x + p)^2 + q$. (c) Hence, or otherwise, state the coordinates of the turning point on the graph of $y = h(x)$. (d) Sketch the graph of $y = h(x) + 3$, showing clearly the coordinates of the turning point and the y -axis intercept.	3 3 1 2

2015 P2	<p>2. Functions f and g are defined on suitable domains by $f(x) = 10 + x$ and $g(x) = (1 + x)(3 - x) + 2$.</p> <p>(a) Find an expression for $f(g(x))$. 2</p> <p>(b) Express $f(g(x))$ in the form $p(x + q)^2 + r$. 3</p> <p>(c) Another function h is given by $h(x) = \frac{1}{f(g(x))}$. What values of x cannot be in the domain of h? 2</p>
2016 P1	<p>12. The functions f and g are defined on \mathbb{R}, the set of real numbers by $f(x) = 2x^2 - 4x + 5$ and $g(x) = 3 - x$.</p> <p>(a) Given $h(x) = f(g(x))$, show that $h(x) = 2x^2 - 8x + 11$. 2</p> <p>(b) Express $h(x)$ in the form $p(x + q)^2 + r$. 3</p>

Pre 2000 – Completing the Square

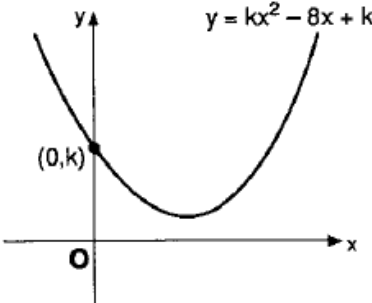
1	<p>Show that $x^2 + 8x + 18$ can be written in the form $(x + a)^2 + b$. Hence or otherwise find the coordinates of the turning point of the curve with equation $y = x^2 + 8x + 18$. 3</p>
2	<p>Express $f(x) = (2x - 1)(2x + 5)$ in the form $a(x + b)^2 + c$. 3</p>
3	<p>(a) Show that the function $f(x) = 2x^2 + 8x - 3$ can be written in the form $f(x) = a(x + b)^2 + c$ where a, b and c are constants.</p> <p>(b) Hence, or otherwise, find the coordinates of the turning point of the function f. 3, 1</p>
4	<p>(a) Show that $f(x) = 2x^2 - 4x + 5$ can be written in the form $f(x) = a(x + b)^2 + c$. 3</p> <p>(b) Hence write down the coordinates of the stationary point of $y = f(x)$ and state its nature. 2</p>
5	<p>(a) Express $7 - 2x - x^2$ in the form $a - (x + b)^2$ and write down the values of a and b. 2</p> <p>(b) State the maximum value of $7 - 2x - x^2$ and justify your answer. 2</p>
6	<p>Express $x^2 + 6x + 11$ in the form $(x + a)^2 + b$ and hence state the maximum value of $\frac{1}{x^2 + 6x + 11}$. 4</p>

Using the Discriminant

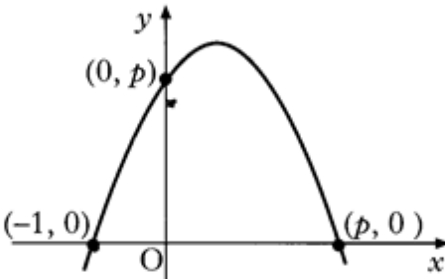
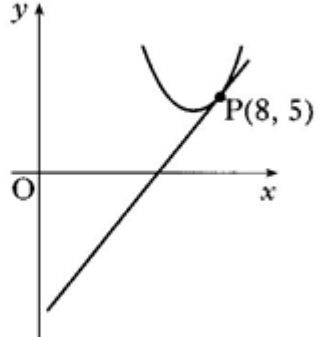
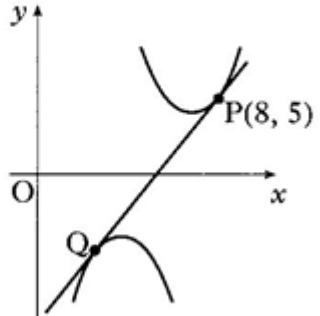
2001 P1	2. For what value of k does the equation $x^2 - 5x + (k + 6) = 0$ have equal roots?	3
2002 P2	9. Show that the equation $(1 - 2k)x^2 - 5kx - 2k = 0$ has real roots for all integer values of k .	5
2003 P1	7. Show that the line with equation $y = 2x + 1$ does not intersect the parabola with equation $y = x^2 + 3x + 4$.	5
2004 P2	3. Prove that the roots of the equation $2x^2 + px - 3 = 0$ are real for all values of p .	4
2006 P2	2. Find the value of k such that the equation $kx^2 + kx + 6 = 0$, $k \neq 0$, has equal roots.	4
2007 P1	4. Find the range of values of k such that the equation $kx^2 - x - 1 = 0$ has no real roots.	4
2008 SP2 P2	2. Prove that the roots of the equation $2x^2 + px - 3 = 0$ are real for all values of p .	4
2015 OLD P1	24. Find the range of values for k such that $kx^2 + 3x + 9k = 0$ has real roots.	4
2015 SP P1	4. Given that $2x^2 + px + p + 6 = 0$ has no real roots, find the range of values for p , where $p \in \mathbb{R}$.	4
2015 EP P2	3. Find the value of p such that the equation $x^2 + (p + 1)x + 9 = 0$ has no real roots.	4
2016 P2	2. Find the range of values for p such that $x^2 - 2x + 3 - p = 0$ has no real roots.	3
2017 P1	4. Find the value of k for which the equation $x^2 + 4x + (k - 5) = 0$ has equal roots.	3

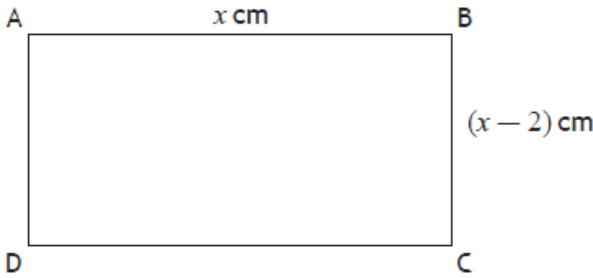
Pre 2000 Questions - Using the Discriminant

1	(i) Write down the condition for the equation $ax^2 + bx + c = 0$ to have no real roots.	1
	(ii) Hence or otherwise show that the equation $x(x + 1) = 3x - 2$ has no real roots.	2
2	For what value of a does the equation $ax^2 + 20x + 40 = 0$ have equal roots?	2
3	The roots of the equation $(x - 1)(x + k) = -4$ are equal. Find the values of k .	5
4	Find the values of k for which the equation $2x^2 + 4x + k = 0$ has real roots.	2

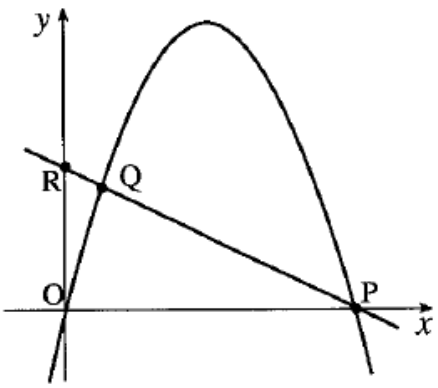
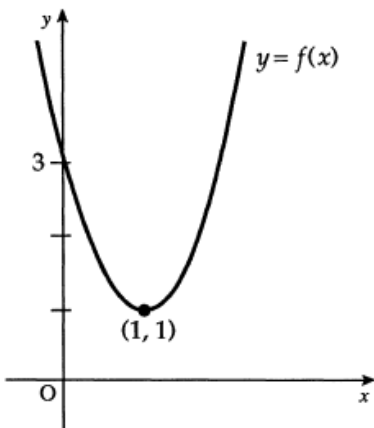
5	Calculate the least positive integer value of k so that the graph of $y = kx^2 - 8x + k$ does not cut or touch the x -axis.		4
6	Given that k is a real number, show that the roots of the equation $kx^2 + 3x + 3 = k$ are always real numbers.		5
7	Show that the roots of the equation $(k-2)x^2 - (3k-2)x + 2k = 0$ are real.		4
8	A curve has equation $y = 2x^3 + 3x^2 + 4x - 5$. Prove that this curve has no stationary points.		5

Extended Questions

2001 P2	<p>11. The diagram shows a sketch of a parabola passing through $(-1, 0)$, $(0, p)$ and $(p, 0)$.</p> <p>(a) Show that the equation of the parabola is $y = p + (p-1)x - x^2$.</p> <p>(b) For what value of p will the line $y = x + p$ be a tangent to this curve?</p>		3 3
2006 P2	<p>3. The parabola with equation $y = x^2 - 14x + 53$ has a tangent at the point $P(8, 5)$.</p> <p>(a) Find the equation of this tangent.</p> <p>(b) Show that the tangent found in (a) is also a tangent to the parabola with equation $y = -x^2 + 10x - 27$ and find the coordinates of the point of contact Q.</p>	 	4 5

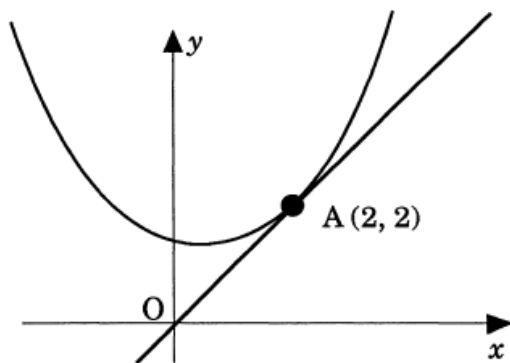
2012 P2	<p>1. Functions f and g are defined on the set of real numbers by</p> <ul style="list-style-type: none"> • $f(x) = x^2 + 3$ • $g(x) = x + 4$. <p>(a) Find expressions for:</p> <p>(i) $f(g(x))$;</p> <p>(ii) $g(f(x))$.</p> <p>(b) Show that $f(g(x)) + g(f(x)) = 0$ has no real roots.</p>	<p>3</p> <p>3</p>
2014 P2	<p>3. Functions f and g are defined on suitable domains by</p> $f(x) = x(x - 1) + q \text{ and } g(x) = x + 3.$ <p>(a) Find an expression for $f(g(x))$.</p> <p>(b) Hence, find the value of q such that the equation $f(g(x)) = 0$ has equal roots.</p>	<p>2</p> <p>4</p>
2015 P1	<p>8. ABCD is a rectangle with sides of lengths x centimetres and $(x - 2)$ centimetres, as shown.</p>  <p>If the area of ABCD is less than 15 cm^2, determine the range of possible values of x.</p>	<p>4</p>
2017 P2	<p>8. Sequences may be generated by recurrence relations of the form $u_{n+1} = k u_n - 20$, $u_0 = 5$ where $k \in \mathbb{R}$.</p> <p>(a) Show that $u_2 = 5k^2 - 20k - 20$.</p> <p>(b) Determine the range of values of k for which $u_2 < u_0$.</p>	<p>2</p> <p>4</p>

Pre 2000 Questions - Extended Questions

1	<p>The parabola shown in the diagram has equation $y = 4x - x^2$ and intersects the x-axis at the origin and P.</p>  <p>(a) Find the coordinates of the point P. 2</p> <p>(b) R is the point (0, 2). Find the equation of PR. 2</p> <p>(c) The line and the parabola also intersect at Q. Find the coordinates of Q. 4</p>
2	<p>The diagram shows a sketch of the parabola $y = f(x)$.</p>  <p>(a) Copy the sketch of $y = f(x)$. On your diagram, draw the parabola with equation $y = -f(x) + 3$. (4)</p> <p>(b) State the values of x for which $3 - f(x) \geq 0$. (2)</p> <p>(c) If $g(x) = 3 - f(x)$, express $g(x)$ in terms of x. (3)</p>
3	<p>(a) $f(x) = 2x + 1$, $g(x) = x^2 + k$, where k is a constant.</p> <p>(i) Find $g(f(x))$. (2)</p> <p>(ii) Find $f(g(x))$. (2)</p> <p>(b) (i) Show that the equation $g(f(x)) - f(g(x)) = 0$ simplifies to $2x^2 + 4x - k = 0$. (2)</p> <p>(ii) Determine the nature of the roots of this equation when $k = 6$. (2)</p> <p>(iii) Find the value of k for which $2x^2 + 4x - k = 0$ has equal roots. (3)</p>

4

- (a) The point $A(2, 2)$ lies on the parabola $y = x^2 + px + q$.
Find a relationship between p and q .



(1)

- (b) The tangent to the parabola at A is the line $y = x$. Find the value of p .
Hence find the equation of the parabola.

(6)

- (c) Using your answers for p and q , find the value of the discriminant of $x^2 + px + q = 0$. What feature of the above sketch is confirmed by this value?

(2)

5

Diagram 1 shows a rectangular plate of transparent plastic moulded into a parabolic shape and pegged to the ground to form a cover for growing plants. Triangular metal frames are placed over the cover to support it and prevent it blowing away in the wind.

Diagram 2 shows an end view of the cover and the triangular frame related to the origin O and axes Ox and Oy . (All dimensions are given in centimetres.)

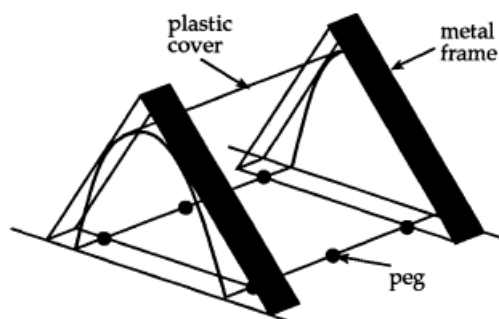


Diagram 1

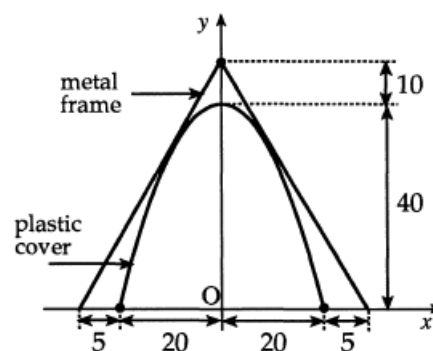


Diagram 2

- (a) Show that the equation of the parabolic end is $y = 40 - \frac{x^2}{100}$, $-20 \leq x \leq 20$.
- (b) Show that the triangular frame touches the cover without disturbing the parabolic shape.

(4)

(7)